09.03.2006.

21:00
21:16
21:19
21:22
21:27

1. $x_{n+1} = x_n + \left(x_n - \frac{1}{3}\right)^2 \qquad n \geq 1,\ x_1 = 0$

$x_n < \frac{1}{3}$

$\frac{1}{9} + \left(\frac{1}{9} - \frac{1}{3}\right)^2$

$\frac{1}{9} + \left(\frac{1-3}{9}\right)^2 = \frac{1}{9} + \frac{4}{81} = \frac{9+4}{81} = \frac{13}{81}$

$x_1 = 0 \quad x_2 = \frac{1}{9} \quad x_3 = \frac{13}{81}$

МОНОТОНОСТ (1)

Низ x_n је монотоно растући $x_n < x_{n+1}$

1. $n=1$ $\quad x_1 < x_2$

$0 < \frac{1}{9}$ T

2. пп да тврђење важи за пр. бр. n

$x_n < x_{n+1}$

3. докажимо да важи за $n+1$

$x_{n+1} < x_{n+2}$

$x_n + \left(x_n - \frac{1}{3}\right)^2 < x_{n+1} + \left(x_{n+1} - \frac{1}{3}\right)^2$

$x_n - x_{n+1} + x_n^2 - \frac{2}{3}x_n + \cancel{\frac{1}{9}} - x_{n+1}^2 + \frac{2}{3}x_{n+1} - \cancel{\frac{1}{9}} < 0$

$x_n^2 + \frac{1}{3}x_n - x_{n+1}^2 - \frac{1}{3}x_{n+1} < 0$

$x_n\left(x_n + \frac{1}{3}\right) - x_{n+1}\left(x_{n+1} + \frac{1}{3}\right) < 0$

$x_n\left(x_n + \frac{1}{3}\right) < x_{n+1}\left(x_{n+1} + \frac{1}{3}\right)$

$x_n < x_{n+1}$ I.H. $\qquad x_n + \cancel{\frac{1}{3}} < x_{n+1} + \cancel{\frac{1}{3}}$

$x_n < x_{n+1}$ I.H.

$\Downarrow$

$x_{n+1} < x_{n+2}$

$\overset{М.И.}{\Longrightarrow}$ Низ x_n је монотоно раст.

ОГРАНИЧЕНОСТ (2)

Низ x_n је ограничен одозго $x_n < \frac{1}{3}$

1. $n=1$ $\quad x_1 < \frac{1}{3}$

$0 < \frac{1}{3}$ T

2. пп. да тврђење важи за свак пр. бр n

$x_n < \frac{1}{3}$

3. докажимо да важи за $n+1$

$x_{n+1} < \frac{1}{3}$

$x_n + \left(x_n - \frac{1}{3}\right)^2 < \frac{1}{3}$

$x_n + x_n^2 - \frac{2}{3}x_n + \frac{1}{9} - \frac{1}{3} < 0$

$x_n^2 + \frac{1}{3}x_n - \frac{2}{9} < 0$

$\underbrace{\left(x - \frac{1}{3}\right)}_{<0\ (I.H)}\underbrace{\left(x + \frac{2}{3}\right)}_{>0\ (1)\ и\ x_1 = 0,\ n \geq 1} < 0$

$x_n = \frac{-3 \pm \sqrt{9+72}}{18} = \frac{-3 \pm 9}{18}$

$\frac{1}{3} \quad -\frac{2}{3}$

$\overset{М.И.}{\Longrightarrow}$ Низ x_n је ограничен одозго $x_n < \frac{1}{3}$

(1) $\wedge$ (2) $\overset{T}{\Longrightarrow}$ низ x_n је КОНВЕРГЕНТАН и $\exists \lim\limits_{n\to\infty} x_n = x$

$\lim\limits_{n\to\infty} x_{n+1} = x_n + \left(x_n - \frac{1}{3}\right)^2$

$\cancel{x} = \cancel{x} + \left(x - \frac{1}{3}\right)^2$

$0 = x^2 - \frac{2}{3}x + \frac{1}{9}$

$9x - 6x + 1 = 0$

$x = \frac{6 \pm \sqrt{36-36}}{18}$

$x = \frac{1}{3}$

$$\boxed{\lim_{n\to\infty} x_n = \frac{1}{3}}$$

2. $\lim\limits_{n\to\infty}\left(\sqrt[3]{n^3+n^2} - n\right) =$

$$= \lim_{n\to\infty} \frac{\left(\sqrt[3]{n^3+n^2} - n\right)\left(\sqrt[3]{(n^3+n^2)^2} + \sqrt[3]{n^3(n^3+n^2)} + \sqrt[3]{(n^3)^2}\right)}{\sqrt[3]{n^6+2n^5+n^4} + \sqrt[3]{n^6+n^5} + \sqrt[3]{n^6}} = \lim_{n\to\infty} \frac{\cancel{n^3} + \cancel{n^2} - \cancel{n^3}}{\cancel{n^2}\left(\sqrt[3]{1+\frac{2}{n}+\frac{1}{n^4}} + \sqrt[3]{1+\frac{1}{n}} + 1\right)} = \frac{1}{3}$$

3. $\sum_{n=1}^{\infty} \left(1-\cos\frac{1}{n}\right)\cdot\frac{1}{n^p} \qquad p\in\mathbb{R}$

$a_n = \left(1-\cos\frac{1}{n}\right)\cdot\frac{1}{n^p} = 2\cdot\sin^2\frac{1}{2n}\cdot\frac{1}{n^p} \sim 2\cdot\frac{1}{4n^2}\cdot\frac{1}{n^p} = \frac{1}{2}\cdot\frac{1}{n^{2+p}} = \frac{1}{2}\cdot b_n$

$\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{1}{2}$ и $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{n^{2+p}}$ — КВ $2+p>1 \quad p>-1$

ДВ $2+p\leq 1 \quad p\leq -1$

$\xRightarrow{\text{п.к.}} \sum_{n=1}^{\infty} a_n \quad \boxed{\text{КВ} \quad p>-1}$

4. $\sum_{n=1}^{\infty}\frac{(n+1)^{n^2}}{n^{n^2}\cdot 2^n}$

$\left(\frac{n+1}{n}\right)^n\cdot\frac{1}{2} = \left(1+\frac{1}{n}\right)^n\cdot\frac{1}{2} = \frac{e}{2}$

$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{\sqrt[n]{(n+1)^{n^2}}}{\sqrt[n]{n^{n^2}\cdot 2^n}} = \lim_{n\to\infty}\frac{(n+1)^n}{2\cdot n^n} = \frac{1}{2}\lim_{n\to\infty}\frac{\left[n\left(1+\frac{1}{n}\right)\right]^n}{n^n} = \frac{1}{2}\lim_{n\to\infty}\frac{\cancel{n^n}\left(1+\frac{1}{n}\right)^n}{\cancel{n^n}} =$

$= \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \frac{e}{2} > 1$

???

$\xRightarrow{\text{К.К.}} \sum_{n=1}^{\infty} a_n \quad \text{ДВ}$

5. $\sum_{n=1}^{\infty}(-1)^{n-1}\, n^2\cdot\sin\frac{\pi}{2^n}$

$|a_n| = \left|(-1)^{n-1}\, n^2\sin\frac{\pi}{2^n}\right| = n^2\sin\frac{\pi}{2^n} \sim n^2\cdot\frac{\pi}{2^n} = \frac{\pi}{2^n}\cdot\frac{1}{n^{-2}} = \frac{\pi}{2^n}\cdot b_n$

$\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\pi}{2^n} = 0$

08.03.2007.

21:33
21:36
21:47
22:09

1. $$\lim_{n\to\infty}\left(\frac{2\sqrt{n}-1}{2\sqrt{n}}\right)^{n(\sqrt{4n+1}-2\sqrt{n})} =$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{2\sqrt{n}}{-1}}\right)^{\frac{2\sqrt{n}}{-1}}\right]^{-\frac{1}{2\sqrt{n}}\cdot\frac{\sqrt{n}^2(4n+1-4n)}{(\sqrt{4n+1}+2\sqrt{n})}} = e^{-\frac{1}{2}\lim_{n\to\infty}\frac{-\sqrt{n}}{2\sqrt{n}(\sqrt{1+\frac{1}{4n}}+1)}} = e^{-\frac{1}{2}\cdot\frac{1}{2\cdot 2}} = e^{-\frac{1}{8}}$$

2. $a_1 = 1 \qquad a_{n+1} = \sqrt[3]{2a_n+4} \quad (n\in N)$

$a_1 = 1 \qquad a_2 = \sqrt[3]{6} \qquad a_3 = \sqrt[3]{16} \qquad a_n \nearrow$

МОНОТОНОСТ (1)

Низ a_n је монотоно растући $a_n < a_{n+1}$

1. $n=1$ $\quad a_1 < a_2$

$1 < \sqrt[3]{6}$ T

2. п.п. да тврђење важи за природан бр n

$a_n < a_{n+1}$

3. докажимо да важи за $n+1$

$a_{n+1} < a_{n+2}$

$\sqrt[3]{2a_n+4} < \sqrt[3]{2a_{n+1}+4} \,/^3$

$2a_n - 2a_{n+1} < 0$

$2\underbrace{(a_n - a_{n+1})}_{<0\,(2.\,\text{н})} < 0$

$\overset{\text{М.И.}}{\Longrightarrow}$ Низ a_n је монотоно растући

Ако је a_n конвергентан онда $\exists_1 \lim_{n\to\infty} a_n = a$

$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\sqrt[3]{2a_n+4}$

$a = \sqrt[3]{a\cdot 2+4} \,/^3$

$a^3 = 2a+4$

$a^3 - 2a - 4 = 0$

$(a-2)(a^2+2a+2) = 0$

$a = 2$ - -

$(a^3-2a-4):(a-2) = a^2+2a+2$

$\underline{-a^3 \mp 2a^2}$

$2a^2-2a-4$

$\underline{-2a^2 \mp 4a}$

$2a-4$

a^2+2a+2

$\underline{-2 \pm \sqrt{\ }}$

ОГРАНИЧЕНОСТ (2)

Низ a_n је ограничен одозго $a_n < 2$

1. $n=1$ $\quad a_1 < 2$

$1 < 2$ T

2. пп да тврђење важи за пр. бр. n

$a_n < 2$

3. докажимо да важи за $n+1$

$a_{n+1} < 2$

$\sqrt[3]{2a_n+4} < 2 \,/^3$

$2a_n+4 < 8$

$2a_n < 4$

$a_n < 2$

$\overset{\text{М.И.}}{\Longrightarrow}$ Низ a_n је ограничен одозго

Из (1) и (2) $\overset{\text{Т}}{\Longrightarrow}$ низ a_n је конвергентан и $\boxed{\lim_{n\to\infty} a_n = 2}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)\cdot 3^n \cdot p^n}{n^2+3}$

$$a_n = \left|\frac{(n+2)3^n \cdot p^n}{n^2+3}\right| = \frac{(n+2)3^n \cdot |p|^n}{n^2+3}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{(n+2)\cdot 3^n \cdot |p|^n}{n^2+3}} = \lim_{n\to\infty} \frac{3\cdot |p| \cdot \sqrt[n]{n(1+\frac{2}{n})}}{\sqrt[n]{n^2(1+\frac{3}{n^2})}} = \lim_{n\to\infty} \frac{3\cdot|p|\cdot \sqrt[n]{n}\cdot\sqrt[n]{1+\frac{2}{n}}}{(\sqrt[n]{n})^2 \cdot \sqrt[n]{1+\frac{3}{n^2}}} = 3|p|$$

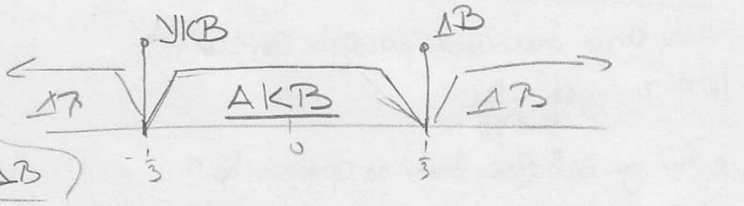
$\xRightarrow{\text{Ко.К.}}$ КВ $3|p|<1$ $\quad |p|<\frac{1}{3}$ $\quad p\in(-\frac{1}{3},\frac{1}{3})$

ДВ $3|p|>1$ $\quad |p|>\frac{1}{3}$ $\quad p\in(-\infty,-\frac{1}{3})\cup(\frac{1}{3},\infty)$

? $3|p|=1$ $\quad p=\pm\frac{1}{3}$

$\displaystyle\sum_{n=1}^{\infty} a_n$ — АКВ $p\in(-\frac{1}{3},\frac{1}{3})$

не КВА $p\in(-\infty,-\frac{1}{3})\cup(\frac{1}{3},\infty)$ ДВ



1) $\displaystyle\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{(n+2)\cdot 3^n |p|^n}{n^2+3}$

$p>\frac{1}{3}$ $\quad \displaystyle\lim_{n\to\infty} \frac{(n+2)\cdot 3^n \cdot p^n}{n^2+3} = \frac{(3p)^n \cdot n(1+\frac{2}{n})}{n^2(1+\frac{3}{n^2})} = +\infty \neq 0$

$\Rightarrow a_n \not\to 0 \quad (n\to\infty)$

$p<-\frac{1}{3}$ $\quad \lim_{n\to\infty} a_n$ не постоји $\Rightarrow a_n \not\to 0 \quad (n\to\infty)$

$\Longrightarrow p\in(-\infty,-\frac{1}{3})\cup(\frac{1}{3},\infty)$ ДВ

$p=\frac{1}{3}$ $\quad \displaystyle\sum_{n=1}^{\infty} \frac{(n+2)\cdot 3^n}{(n^2+3)\cdot 3^n}$

$a_n = \frac{(n+2)\cdot \cancel{3^n}}{(n^2+3)\cdot \cancel{3^n}} \sim \frac{1}{n}$ $\quad p=1$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ ДВ

$\xRightarrow{\text{П.К}}$ $\displaystyle\sum_{n=1}^{\infty} a_n$ $\quad p=\frac{1}{3}$ ДВ

$p=-\frac{1}{3}$

$$\sum_{n=1}^{\infty} \frac{(n+2)\cdot \cancel{3^n}\cdot(-1)^n}{(n^2+3)\cdot \cancel{3^n}} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{n+2}{n^2+3}$$

$|a_n| = \left|(-1)^n \frac{n+2}{n^2+3}\right| = \frac{n+2}{n^2+3} \sim \frac{1}{n} \Rightarrow \displaystyle\sum_{n=1}^{\infty}|a_n|$ ДВ

1. $\displaystyle\lim_{n\to\infty} \frac{n+2}{n^2+3} = 0$

2. $|a_n|\downarrow$

$f(x) = \frac{n+2}{n^2+3}$ $\quad f'(x) = \frac{n^2+3-(n+2)\cdot n}{(n^2+3)^2} = \frac{\cancel{n^2}+3-\cancel{n^2}-2n}{(n^2+3)^2} = \frac{3-2n}{(n^2+3)^2}$

$3-2n<0$

$f'(x)<0$

$\xRightarrow{\text{Л.К.}}$ У.КВ

13.03.2008.

1. $a_1 = 1 \qquad a_{n+1} = \sqrt{a_n + 2}$

$a_1 = 1 \quad a_2 = \sqrt{3}$

МОНОТОНОСТ (1)

Низ a_n је монотоно растући $a_n < a_{n+1}$

1. $n = 1 \qquad a_1 < a_2$

$1 < \sqrt{3} \quad T$

2. пп да тврђење важи за пр бр n

$a_n < a_{n+1}$

3. доказимо да важи за $n+1$

$a_{n+1} < a_{n+2}$

$\sqrt{a_n + 2} < \sqrt{a_{n+1} + 2} \;/^2$

$a_n + \cancel{2} < a_{n+1} + \cancel{2}$

$a_n < a_{n+1}$

$\xRightarrow{М.И.}$ Низ a_n је монотоно растући

Ако је a_n КОНВЕРГЕНТАН $\exists \lim_{n\to\infty} a_n = a$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{a_n + 2}$

$a = \sqrt{a+2} \;/^2$

$a^2 = a + 2$

$a^2 - a - 2 = 0$

$\frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$

$\lim_{n\to\infty} a_n = 2$

22:12
22:27
22:36
22:49

ОГРАНИЧЕНОСТ (2)

Низ a_n је ограничен одозго $a_n < 2$

1) $n = 1 \qquad a_1 < 2$

$1 < 2 \quad T$

2) пп да тврђење важи за пр. бр n

$a_n < 2$

3. доказимо да важи за $n+1$

$a_{n+1} < 2$

$\sqrt{a_n + 2} < 2 \;/^2$

$a_n + 2 < 4$

$a_n < 2$

$\xRightarrow{М.И.}$ Низ a_n је ограничен одозго

Из (1) и (2) $\xRightarrow{T}$ низ a_n је КОНВЕРГЕНТАН и $\lim_{n\to\infty} a_n = 2$

2. $$\lim_{n\to\infty}\left(\frac{n^2-n+1}{n^2+n+1}\right)^{\sqrt[3]{n^2}(\sqrt[3]{n+1}-1)} = \lim_{n\to\infty}\left(\frac{n^2+n+1-2n}{n^2+n+1}\right)^{\sqrt[3]{n^2}(\sqrt[3]{n+1}-1)} = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{n^2+n+1}{-2n}}\right)^{\frac{n^2+n+1}{-2n}}\right]^{\frac{-2n}{n^2+n+1}\cdot\sqrt[3]{n^2}(\sqrt[3]{n+1}-1)} =$$

$$= e^{\lim_{n\to\infty} \frac{-2\,\cancel{n^{5/3}}}{\cancel{n^2}\left(1+\frac{1}{n}+\frac{1}{n^2}\right)} \cdot \cancel{n^{1/3}}\left(\sqrt[3]{1+\frac{1}{n}} - \frac{1}{\sqrt[3]{n}}\right)} = e^{-2}$$

3. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(\frac{a+1}{a^2+1}\right)^n$$

$$|a_n| = \left|\frac{(-1)^n}{n}\cdot\left(\frac{a+1}{a^2+1}\right)^n\right| = \frac{1}{n}\cdot\frac{(|a+1|)^n}{(a^2+1)^n}$$

$$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{1}{\cancel{\sqrt[n]{n}}_{1}}\cdot\frac{\sqrt[n]{|a+1|^n}}{\sqrt[n]{(a^2+1)^n}} = \frac{|a+1|}{a^2+1}$$

$\xRightarrow{К.К} \sum_{n=1}^{\infty}|a_n|$

КВ $\qquad \frac{|a+1|}{a^2+1} < 1 \qquad a \in (-\infty, 0) \cup (1, \infty)$

ДВ $\qquad \frac{|a+1|}{a^2+1} > 1 \qquad a \in (0, 1)$

? $\qquad \frac{|a+1|}{a^2+1} = 1 \qquad a = 0$

$a = 1$

$|a+1| < a^2+1$

$a+1 < a^2+1 \quad \vee \quad -a-1 = a^2+1$

$a^2 - a > 0 \qquad a^2+a+2 = 0$

$a(a-1) = 0$

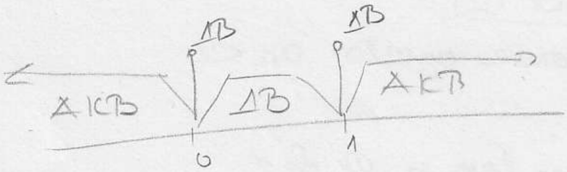
$a = 0 \quad a = 1$

$+ + \quad 0 \quad - \quad 1 \quad + +$

АКВ $a \in (-\infty, 0) \cup (1, \infty)$

$\Rightarrow \sum_{n=1}^{\infty} a_n$ не КВА $a \in (0, 1)$

1. $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{n} \cdot \left(\overset{>1}{\frac{a+1}{a^2+1}}\right)^n = \infty \neq 0$



$\Rightarrow$ $\boxed{p \in (0,1) \quad \sum_{n=1}^{\infty} a_n \ \text{ΛВ}}$

<u>$a = 0$</u>

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n} \quad p = 1$ ΛВ

$\xrightarrow{n.k} \sum_{n=1}^{\infty} a_n \quad a = 0$ ΛВ

$a = 1$

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n} \cdot$ ΛВ $\Rightarrow \sum_{n=1} a_n \quad a=0$ ΛВ

1. $\sum_{n=1}^{\infty} \left(\sqrt[3]{n+2} - \sqrt[3]{n-2}\right)(n^2+1)^p$

$$a_n = \frac{(\cancel{n}+2-\cancel{n}+2)(n^2+1)^p}{\sqrt[3]{(n+2)^2}+\sqrt[3]{(n^2-4)}+\sqrt[3]{(n-2)^2}} = \frac{4n^{2p}\left(1+\frac{1}{n^2}\right)^p}{n^{\frac{2}{3}}\left(\sqrt[3]{(1+\frac{2}{n})^2}+\sqrt[3]{1-\frac{4}{n^2}}+\sqrt[3]{(1-\frac{2}{n})^2}\right)} = \frac{4}{3}\cdot\frac{1}{n^{\frac{2}{3}-2p}} = \frac{4}{3}b_n$$

$\lim\limits_{n\to\infty}\frac{a_n}{b_n} = \frac{4}{3}$ и $\sum\limits_{n=1}^{\infty} b_n = \sum\limits_{n=1}^{\infty}\frac{1}{n^{\frac{2}{3}-2p}}$ — КВ $\frac{2}{3}-2p>1 \quad p<-\frac{1}{6}$; ДВ $\frac{2}{3}-2p\le 1 \quad p\ge -\frac{1}{6}$

$\xrightarrow{\text{п.к.}} \sum\limits_{n=1}^{\infty} a_n$ — КВ $p<-\frac{1}{6}$; ДВ $p\ge -\frac{1}{6}$

2. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}-\sqrt[3]{n^2-1}}{n^{2p}}$

$$a_n = \frac{\cancel{n^2}+1-\cancel{n^2}+1}{n^{2p}\cdot n^{\frac{4}{3}}\left(\sqrt[3]{(1+\frac{1}{n^2})^2}+\sqrt[3]{1-\frac{1}{n^4}}+\sqrt[3]{(1-\frac{1}{n^2})^2}\right)} \sim \frac{1}{3}\cdot\frac{1}{n^{2p+\frac{4}{3}}} = \frac{1}{3}b_n$$

$\lim\limits_{n\to\infty}\frac{a_n}{b_n} = \frac{1}{3}$ и $\sum\limits_{n=1}^{\infty} b_n = \sum\limits_{n=1}^{\infty}\frac{1}{n^{2p+\frac{4}{3}}}$ — КВ $2p+\frac{4}{3}>1 \quad p>-\frac{1}{6}$; ДВ $2p+\frac{4}{3}\le 1 \quad p\le -\frac{1}{6}$

$\xrightarrow{\text{п.к.}} \sum\limits_{n=1}^{\infty} a_n$ — КВ $p>-\frac{1}{6}$; ДВ $p\le -\frac{1}{6}$

5. $\sum_{n=1}^{\infty} \frac{2\cdot 6\cdots(4n-2)}{2^{pn}(2n+1)!!} \quad p\ne 1$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\dfrac{\cancel{2\,6\cdots(4n-2)}(4n+2)}{2^{p(n+1)}(2n+3)!!}}{\dfrac{\cancel{2\,6\cdots(4n-2)}}{2^{pn}\cdot(2n+1)!!}} = \lim_{n\to\infty}\frac{\cancel{2^{pn}}\cdot\cancel{(2n+1)!!}(4n+2)}{\cancel{2^{pn}}\cdot 2^p(2n+3)\cancel{(2n+1)!!}} =$$

$$= \frac{1}{2^p}\cdot\lim_{n\to\infty}\frac{\cancel{n}(4+\frac{2}{n})}{\cancel{n}(2+\frac{3}{n})} = \frac{4}{2^{p+1}}$$

$\xrightarrow{\text{Д.К.}} \sum\limits_{n=1}^{\infty} a_n$ — $\boxed{\text{КВ } \frac{4}{2^{p+1}}<1 \quad p>1}$; $\boxed{\text{ДВ } \frac{4}{2^{p+1}}>1 \quad p<1}$

6. $\sum_{n=1}^{\infty} \frac{(6n-4)(6n-7)\dots 8\cdot 5\cdot 2}{3^{pn}(2n)!}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(6n+2)(6n-1)\cancel{(6n-4)(6n-7)\dots 8\cdot 5\cdot 2}}{\cancel{3^{pn}}\cdot 3^p\cdot(2n+2)!}}{\frac{\cancel{(6n-4)(6n-7)\dots 8\cdot 5\cdot 2}}{\cancel{3^{pn}}(2n)!}} = \lim_{n\to\infty} \frac{\cancel{(2n)!}(36n^2+6n-2)}{3^p\cdot(2n+2)(2n+1)\cdot\cancel{(2n)!}} =$$

$4n^2+6n+2$

$$= \frac{1}{3^p}\cdot\lim_{n\to\infty} \frac{\cancel{n^2}(36+\frac{6}{n}-\frac{2}{n^2})}{\cancel{n^2}(4+\frac{6}{n}+\frac{2}{n^2})} = \frac{1}{3^p}\cdot 9 = \frac{1}{3^{p-2}}$$

$\xRightarrow{\text{д.к.}}$ $\sum_{n=1}^{\infty} a_n$ — КВ $\frac{1}{3^{p-2}} < 1$ $p>2$ — ДВ $\frac{1}{3^{p-2}} > 1$ $p<2$

$p=2$

7. $\sum_{n=1}^{\infty} (\sqrt{n+1}-\sqrt{n})^p \cdot \ln\frac{2n+1}{2n-1}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\ln\frac{2n+3}{2n+1}}{\ln\frac{2n+1}{2n-1}}\cdot\left(\frac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+1}-\sqrt{n}}\right)^p = 1$$

$$\frac{\ln(2n+3)-\ln(2n+1)}{\ln(2n+1)-\ln(2n-1)} = \frac{\frac{2}{2n+3}-\frac{2}{2n+1}}{\frac{2}{2n+1}-\frac{2}{2n-1}} = \frac{\frac{\cancel{4n}+2-\cancel{4n}-6}{(2n+3)\cancel{(2n+1)}}}{\frac{\cancel{4n}-2-\cancel{4n}-2}{\cancel{(2n+1)}(2n-1)}} =$$

$$= \frac{\cancel{-4}(2n-1)}{\cancel{-4}(2n+3)} = \frac{2n-1}{2n+3} = \frac{2}{2} = 1$$

$$= \lim_{n\to\infty}\left(\frac{\frac{\cancel{n}+2-\cancel{n}-1}{\sqrt{n+2}+\sqrt{n+1}}}{\frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}+\sqrt{n}}}\right)^p = \lim_{n\to\infty}\left(\frac{\cancel{\sqrt{n}}(\sqrt{1+\frac{1}{n}}+1)}{\cancel{\sqrt{n}}(\sqrt{1+\frac{2}{n}}+\sqrt{1+\frac{1}{n}})}\right)^p = 1^p$$

$$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{\cancel{n}+1-\cancel{n}}{\sqrt{n}(\sqrt{1+\frac{1}{n}}+1)}\right)^p} = \left(\frac{1}{1\cdot 1}\right)^p = 1^p$$

$$a_n = (\sqrt{n+1}-\sqrt{n})^p\cdot\ln\frac{2n+1}{2n-1} = \frac{n+1-n}{(\sqrt{n+1}+\sqrt{n})^p}\cdot\ln\frac{2n-1+2}{2n-1} = \frac{1}{\sqrt{n^p}(\sqrt{1+\frac{1}{n}}+1)^p}\cdot\ln\left(1+\frac{1}{\frac{2n-1}{2}}\right)\sim$$

$$\sim \frac{1}{2^p\cdot n^{\frac{p}{2}}}\cdot\frac{2}{2n-1}\sim\frac{1}{2^p\cdot n^{\frac{p}{2}}}\cdot\frac{1}{n} = \frac{1}{2^p}\cdot\frac{1}{n^{\frac{p}{2}+1}} = \frac{1}{2^p}\cdot b_n$$

$\cancel{\frac{2}{2n(1-\frac{1}{2n})}}$

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{n^{\frac{p}{2}+1}}$ — КВ $\frac{p}{2}+1>1$ $p>0$ — ДВ $\frac{p}{2}+1\le 1$ $p\le 0$

8. $\sum_{n=2}^{\infty} \sqrt{\ln(1+\frac{2}{n})} \cdot \left(\ln \frac{\sqrt{n}-1}{\sqrt{n}+1}\right)^p$

$$a_n = \sqrt{\ln(1+\frac{2}{n})} \cdot \left[\ln\left(1+\frac{-2}{\sqrt{n}+1}\right)\right]^p \sim \left(\frac{2}{n}\right)^{\frac{1}{2}} \cdot \left(\frac{-2}{\sqrt{n}+1}\right)^p = \frac{-2^{\frac{1}{2}+p}}{n^{\frac{1}{2}} \cdot \left(\sqrt{n}(1+\frac{1}{\sqrt{n}})\right)^p} \sim \frac{-2^{\frac{1}{2}+p}}{n^{\frac{1}{2}+\frac{p}{2}}}$$

$$= \frac{-2^{\frac{1}{2}+p}}{n^{\frac{p+1}{2}}}$$

$\lim_{n\to\infty} \frac{a_n}{b_n} = -2^{\frac{1}{2}+p}$ и $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{p+1}{2}}}$ $\begin{cases} \text{КВ} & \frac{p+1}{2} > 1 \quad p > 1 \\ \text{ДВ} & \frac{p+1}{2} \le 1 \quad p \le 1 \end{cases}$

$\xRightarrow{\text{п.к}} \sum_{n=1}^{\infty} a_n \begin{cases} \text{КВ} & p > 1 \\ \text{ДВ} & p \le 1 \end{cases}$

9. $\sum_{n=1}^{\infty} (1-\cos\frac{1}{n}) \cdot \frac{1}{\sqrt{n^p}}$

$$a_n = (1-\cos\frac{1}{n}) \cdot \frac{1}{\sqrt{n^p}} = 2\cdot\sin^2\frac{1}{2n} \cdot \frac{1}{\sqrt{n^p}} \sim 2 \cdot \frac{1}{4n^2} \cdot \frac{1}{n^{\frac{p}{2}}} = \frac{1}{2} \cdot \frac{1}{n^{2+\frac{p}{2}}}$$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{2}$ и $\sum_{n=1}^{\infty} \frac{1}{n^{2+\frac{p}{2}}}$ $\begin{cases} \text{КВ} & 2+\frac{p}{2} > 1 \quad p > -2 \\ \text{ДВ} & 2+\frac{p}{2} \le 1 \quad p \le -2 \end{cases}$

$\xRightarrow{\text{п.к}} \boxed{\sum_{n=1}^{\infty} a_n \begin{cases} \text{КВ} & p > -2 \\ \text{ДВ} & p \le -2 \end{cases}}$

10. $\sum_{n=1}^{\infty} (1-\cos\frac{1}{\sqrt{n}}) \cdot \frac{1}{n^p}$

$$a_n = (1-\cos\frac{1}{\sqrt{n}}) \cdot \frac{1}{n^p} = 2\sin^2\frac{1}{2\sqrt{n}} \cdot \frac{1}{n^p} \sim 2 \cdot \left(\frac{1}{2\sqrt{n}}\right)^2 \cdot \frac{1}{n^p} = \frac{1}{2} \cdot \frac{1}{n^{p+1}}$$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{2}$ и $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ $\begin{cases} \text{КВ} & p+1 > 1 \quad p > 0 \\ \text{ДВ} & p+1 \le 1 \quad p \le 0 \end{cases}$

$\xRightarrow{\text{п.к}} \boxed{\sum_{n=1}^{\infty} a_n \begin{cases} \text{КВ} & p > 0 \\ \text{ДВ} & p \le 0 \end{cases}}$

11. $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+\sqrt{n+\sqrt{n}}}-\sqrt{n}}{n}\right)^p\cdot tg^2\frac{\pi}{\sqrt[3]{n}}$

$$a_n=\left(\frac{\sqrt{n+\sqrt{n+\sqrt n}}-\sqrt n}{n}\cdot\frac{\sqrt{n+\sqrt{n+\sqrt n}}+\sqrt n}{-\|-}\right)^p\cdot tg^2\frac{\pi}{\sqrt[3]{n}}=$$

$$=\left(\frac{\cancel{n}+\sqrt{n+\sqrt n}-\cancel{n}}{n\cdot\sqrt n\left(\sqrt{1+\frac{\sqrt{n+\sqrt n}}{n}}+1\right)}\right)^p\cdot tg^2\frac{\pi}{\sqrt[3]{n}}=$$

$$=\left(\frac{\cancel{\sqrt n}\cdot\sqrt{1+\sqrt{\frac1n}}}{n\cancel{\sqrt n}\left(\sqrt{1+\sqrt{\frac1n}+\sqrt{\frac{1}{n^3}}}\right)}\right)^p\cdot tg^2\frac{\pi}{\sqrt[3]{n}}\sim\frac{1}{n^p}\cdot\left(\frac{\pi}{\sqrt[3]{n}}\right)^2=\pi^2\cdot\frac{1}{n^{p+\frac23}}$$

$\lim_{n\to\infty}\frac{a_n}{b_n}=\pi^2$ и $\sum_{n=1}^{\infty}\frac{1}{n^{p+\frac23}}$ — КВ $p+\frac23>1$ $p>\frac13$; ДВ $p+\frac23\leq1$ $p\leq\frac13$

$\overset{п.к.}{\Longrightarrow}\sum_{n=1}^{\infty}a_n$ — КВ $p>\frac13$; ДВ $p\leq\frac13$

12. $\sum_{n=1}^{\infty}\frac{e^{1/\sqrt n}-1}{(\sqrt{n+2}-\sqrt n)^p}$

$$a_n=\frac{(e^{1/\sqrt n}-1)\cdot\sqrt{n}^p\left(\sqrt{1+\frac2n}+1\right)^p}{(\cancel{n}+2-\cancel{n})^p}\sim\frac{1}{\sqrt n}\cdot\frac{n^{\frac p2}\cdot\cancel{2^p}}{\cancel{2^p}}=\frac{1}{n^{\frac{1-p}{2}}}$$

$\lim_{n\to\infty}\frac{a_n}{b_n}=1$ и $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1-p}{2}}}$ — КВ $\frac{1-p}{2}>1$ $p<-1$; ДВ $\frac{1-p}{2}\leq1$ $p\geq-1$

$\overset{п.к.}{\Longrightarrow}$ $\boxed{\sum_{n=1}^{\infty}a_n \text{ — КВ } p<-1;\ \text{ДВ } p\geq-1}$

13. $\sum_{n=1}^{\infty}\left(\frac{n^2+pn-3}{n^2+2n+5}\right)^{n^2}$

$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{n^2+pn-3}{n^2+2n+5}\right)^{n^2}}=\lim_{n\to\infty}\left(\frac{n^2+pn-3}{n^2+2n+5}\right)^n=\lim_{n\to\infty}\left(\frac{n^2+2n+5-2n+pn-8}{n^2+2n+5}\right)^n=$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{n^2+2n+5}{n(p-2)-8}}\right)^{\frac{n^2+2n+5}{n(p-2)-8}}\right]^{\frac{n(p-2)-8}{n^2+2n+5}\cdot n}=e^{\lim_{n\to\infty}\frac{n^2(p-2)-8n}{n^2+2n+5}}=$$

$$=e^{\lim_{n\to\infty}\frac{\cancel{n^2}(p-2-\frac8n)}{\cancel{n^2}(1+\frac2n+\frac5{n^2})}}=e^{p-2}$$

K.K.

$$\Longrightarrow \sum_{n=1}^{\infty} a_n \quad \begin{cases} KB & e^{p-2}<1 & p<2 \\ \Delta B & e^{p-2}>1 & p>2 \\ ? & e^{p-2}=1 & p=2 \quad \Delta B \end{cases}$$

$\underline{p=2}$

$$a_n=\left(\frac{n^2+2n-3}{n^2+2n+5}\right)^{n^2}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{n^2+2n+5}{-8}}\right)^{\frac{n^2+2n+5}{-8}}\right]^{\frac{-8n^2}{n^2+2n+5}} = e^{\lim \frac{-8n^2}{\cancel{n^2}(1+\frac{2}{n}+\frac{5}{n^2})}} = e^{-8} \neq 0$$

$$\Longrightarrow p=2 \quad \sum_{n=1}^{\infty} a_n \ \underline{\Delta B}$$

14. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n^2-pn}{3n^2+2n+1}\right)^{n(n+1)}$

$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{3n^2-pn}{3n^2+2n+1}\right)^{n(n+1)}}=\lim_{n\to\infty}\left(\frac{3n^2+2n+1-2n-pn-1}{3n^2+2n+1}\right)^{n+1}=$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{3n^2+2n+1}{-n(2+p)-1}}\right)^{\frac{3n^2+2n+1}{-n(2+p)-1}}\right]^{\frac{(-n(2+p)-1)(n+1)}{3n^2+2n+1}}=$$

$$=e^{\lim_{n\to\infty}\frac{-n^2(2+p)-n(3+p)-1}{3n^2+2n+1}}=e^{\lim_{n\to\infty}\frac{-\cancel{n^2}(2+p+\frac{3+p}{n}+\frac{1}{n^2})}{3\cancel{n^2}(1+\frac{1}{6n}+\frac{1}{3n^2})}}=e^{\frac{-2-p}{3}}$$

K.K.

$$\Longrightarrow \begin{array}{llll} KB & e^{\frac{-2-p}{3}}<1 & \boxed{p>-2 \quad KB} \\ \Delta B & e^{\frac{-2-p}{3}}>1 & \boxed{p<-2 \quad \Delta B} \\ ? & e^{\frac{-2-p}{3}}=1 & \boxed{p=2 \quad \underline{\Delta B}} \end{array}$$

$\underline{p=2}$

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\left(\frac{3n^2-2n}{3n^2+2n+1}\right)^{n(n+1)}=\lim_{n\to\infty}\left[\left(1+\frac{1}{\frac{3n^2+2n+1}{-4n-1}}\right)^{\frac{3n^2+2n+1}{-4n-1}}\right]^{\frac{-(4n+1)\cdot n(n+1)}{3n^2+2n+1}}=e^{-\infty}\neq 0$$

$$\Longrightarrow p=1 \quad \Delta B$$

15. $\sum_{n=1}^{\infty} \frac{p^n}{n \cdot 3^n}$

$|a_n| = \left|\frac{p^n}{n\cdot 3^n}\right| = \frac{|p|^n}{n \cdot 3^n}$

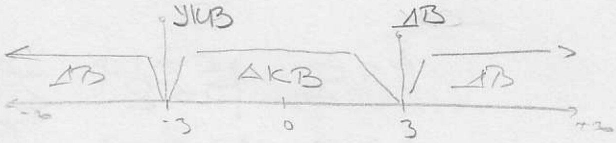
$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{|p|^n}{n\cdot 3^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{|p|^n}}{\underset{1}{\sqrt[n]{n}} \cdot \sqrt[n]{3^n}} = \frac{|p|}{3}$

$\xRightarrow{\text{K.K.}} \sum_{n=1}^{\infty} |a_n|$ КВ $\frac{|p|}{3} < 1$ $p \in (-3, 3)$

ДВ $\frac{|p|}{3} > 1$ $p \in (-\infty, -3) \cup (3, \infty)$

? $\frac{|p|}{3} = 1$ $p = \pm 3$

$\Rightarrow$ $\boxed{\sum_{n=1}^{\infty} a_n \quad \text{АКВ} \quad p \in (-3, 3)}$



1. $p > 3$ $\lim_{n\to\infty} \frac{p^n}{n \cdot 3^n} = \lim_{n\to\infty} \left(\frac{p}{3}\right)^n \frac{1}{n} = \infty \neq 0$ $\Rightarrow$ $p > 3$ ДВ

$p < -3$ $\lim_{n\to\infty} \frac{p^n}{n \cdot 3^n}$ – не постоји $\Rightarrow$ $p < -3$ ДВ

$p = 3$

$\sum_{n=1}^{\infty} \frac{\cancel{3^n}}{n \cdot \cancel{3^n}} = \sum_{n=1}^{\infty} \frac{1}{n}$

п.к.

$a_n = \frac{1}{n} \sim \frac{1}{n}$ $\sum_{n=1}^{\infty} \frac{1}{n}$ ДВ $\xRightarrow{\text{п.к.}}$ $\sum_{n=1}^{\infty} a_n$ ДВ

$p = -3$

$\sum_{n=1}^{\infty} \frac{(-3)^n}{n \cdot 3^n} = \sum_{n=1} \frac{(-1)^n}{n}$

$|a_n| = \left|\frac{(-1)^n}{n}\right| = \frac{1}{n} \sim \frac{1}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ ДВ $\xRightarrow{\text{п.к.}}$ $\sum_{n=1}^{\infty} |a_n|$ ДВ

$\xRightarrow{\text{п.к.}}$ $\sum_{n=1}^{\infty} a_n$ не КВА

1. $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{\cancel{3^n}}{n \cdot \cancel{3^n}} = 0$ ✓

2. $|a_n| \downarrow$ $|a_n| > |a_{n+1}|$

$n < n+1$

$\frac{1}{n} > \frac{1}{n+1}$ ✓

$|a_n| \downarrow$

$\xrightarrow{\text{Л.К.}}$ УКВ

(16) $\sum_{n=1}^{\infty} \frac{2^n p^n}{n^2+1}$

$\frac{(-1/p)^n}{n^2+1} = \frac{-1}{1+1} \sim -\frac{1}{2}$

$\frac{2}{5}$

-1

$|a_n| = \left|\frac{2^n \cdot p^n}{n^2+1}\right| = \frac{2^n \, |p|^n}{n^2+1}$

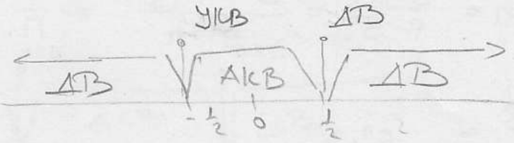
$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{2^n \cdot |p|^n}{n^2+1}} = \lim_{n\to\infty} \frac{\sqrt[n]{2^n} \cdot \sqrt[n]{|p|^n}}{\sqrt[n]{n^2+1}} = \lim_{n\to\infty} \frac{2|p|}{\underbrace{\sqrt[n]{n^2}}_{1} \cdot \sqrt[n]{1+\underbrace{\frac{1}{n^2}}_{0}}} = 2|p|$

$\xRightarrow{\text{K.K.}} \sum_{n=1}^{\infty} |a_n|$ КВ $\quad 2|p| < 1 \quad p \in (-\frac{1}{2}, \frac{1}{2})$

ДВ $\quad 2|p| > 1 \quad p \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$

? $\quad 2|p| = 1 \quad p = \pm\frac{1}{2}$

$\Longrightarrow \boxed{\sum_{n=1}^{\infty} a_n \text{ АКВ } \quad p \in (-\frac{1}{2}, \frac{1}{2})}$



$p > \frac{1}{2} \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2^n \cdot p^n}{n^2+1} = \lim_{n\to\infty} \frac{(2p)^n}{n^2(1+\underbrace{\frac{1}{n^2}}_{0})} = \infty \neq 0 \quad \Rightarrow$ ред ДВ

$p < -\frac{1}{2} \quad \lim_{n\to\infty} a_n$ – не постоји $\quad a_n \underset{n\to\infty}{\not\to} 0 \quad \Rightarrow$ ДВ

$\underline{p = \frac{1}{2}} \quad \sum_{n=1}^{\infty} \frac{1}{n^2+1}$

$a_n = \frac{1}{n^2+1} \sim \frac{1}{n^2} \quad p > 1$

$\lim_{n\to\infty} \frac{a_n}{b_n} = 1 \quad$ и $\sum_{n=1}^{\infty} \frac{1}{n^2}$ ДВ

$\xRightarrow{\text{п.к.}} \sum_{n=1}^{\infty} a_n$ ДВ

$\underline{p = -\frac{1}{2}} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$

$|a_n| = \left|\frac{(-1)^n}{n^2+1}\right| = \frac{1}{n^2+1} \sim \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ ДВ $\xRightarrow{\text{п.к.}} \sum_{n=1}^{\infty} |a_n|$ ДВ

$\Rightarrow \sum_{n=1}^{\infty} a_n$ не КВА (ДВ или УКВ)

1. $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{n^2+1} = 0$ ✓

2. $|a_n| \downarrow$

$n < n+1$

$n^2 < (n+1)^2$

$n^2+1 < (n+1)^2+1$

$\frac{1}{n^2+1} > \frac{1}{(n+1)^2+1}$

$|a_n| > |a_{n+1}|$

$|a_n| \downarrow \xRightarrow{\text{Л.К.}}$ УКВ

(17) $\sum_{n=1}^{\infty} \frac{(n+2)(p+2)^n}{n!}$

$|a_n| = \frac{(n+2)(|p+2|)^n}{n!}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+3)(|p+2|)^{n+1}}{(n+1)!}}{\frac{(n+2)(|p+2|)^n}{n!}} = \lim_{n\to\infty} \frac{(n+3)\cdot(|p+2|)\cdot \cancel{n!}}{(n+2)\cdot(n+1)\cdot \cancel{n!}} = \lim_{n\to\infty} \frac{(n+3)\cdot|p+2|}{n^2+3n+2} = |p+2| \cdot \lim_{n\to\infty} \frac{\cancel{n}(1+\frac{3}{n})}{n^{\cancel{2}}(1+\frac{3}{n}+\frac{2}{n^2})} = 0$

< 1

$\xRightarrow{\text{K.K.}} \sum_{n=1}^{\infty} |a_n|$ КВ $\quad p \in \mathbb{R} \quad \Rightarrow \sum_{n=1}^{\infty} a_n$ АКВ $p \in \mathbb{R}$

(18) $\sum_{n=1}^{\infty} \frac{5^n + 3^n}{n} p^n$

$|a_n| = \frac{5^n+3^n}{n} \cdot |p|^n$

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{5^n+3^n}{n} \cdot |p|^n} = \lim_{n\to\infty} \frac{\sqrt[n]{5^n}\sqrt[n]{(1+(\frac{3}{5})^n)}}{\underbrace{\sqrt[n]{n}}_{1}} \cdot \sqrt[n]{|p|^n} = 5|p|$

$\xRightarrow{\text{K.K.}} \sum_{n=1}^{\infty} |a_n|$ КВ $\quad 5|p| < 1 \quad p \in (-\frac{1}{5}, \frac{1}{5})$

ДВ $\quad 5|p| > 1 \quad p \in (-\infty, -\frac{1}{5}) \cup (\frac{1}{5}, \infty)$

? $\quad 5|p| = 1 \quad p = \pm\frac{1}{5}$

$\Longrightarrow \sum_{n=1}^{\infty} a_n$ АКВ $\quad p \in (-\frac{1}{5}, \frac{1}{5})$

$\underline{p>\frac{1}{5}}$ 1. $\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} \frac{5^n+3^n}{n}\cdot p^n = \lim\limits_{n\to\infty} \frac{5^n\left(1+\left(\frac{3}{5}\right)^n\right)}{n}\cdot p^n = \infty \Rightarrow \underset{n\to\infty}{a_n \not\to 0}$ ДВ

$p<-\frac{1}{5}$ $\lim\limits_{n\to\infty} a_n$ = не постоји $\underset{n\to\infty}{|a_n| \not\to 0}$ ДВ

($5p<-1$)

$$\boxed{\sum_{n=1}^{\infty} a_n \quad \text{ДВ} \quad p\in\left(-\infty,-\frac{1}{5}\right)\cup\left(\frac{1}{5},\infty\right)}$$

$\cancel{p=\frac{1}{5}}$ $\sum\limits_{n=1}^{\infty} \frac{5^n+3^n}{n}\, p^n$

$a_n = \frac{5^n+3^n}{n\cdot 5^n} = \frac{\cancel{5^n}\left(1+\left(\frac{3}{5}\right)^n\right)}{\cancel{5^n}\cdot n} \sim \frac{1}{n}$ $p=1$ ДВ

$p=-\frac{1}{5}$

$\sum\limits_{n=1}^{\infty} = \frac{5^n+3^n}{n\cdot(-1)^n\, 5^n}$

$|a_n| = \left|\frac{5^n+3^n}{(-1)^n\, 5^n\cdot n}\right| = \frac{5^n+3^n}{5^n\cdot n} \sim \frac{1}{n}$ $p=1$ $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ ДВ

$\overset{\text{п.к.}}{\Longrightarrow} \sum\limits_{n=1}^{\infty} |a_n|$ не КВА

1. $\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} \frac{5^n+3^n}{5^n\cdot n} = \lim\limits_{n\to\infty} \frac{\cancel{5^n}\left(1+\left(\frac{3}{5}\right)^n\right)}{\cancel{5^n}\cdot n} = 0$ ✓

2. $|a_n| \downarrow$

$f(x) = \frac{5^x+3^x}{5^x\cdot x} = \frac{\cancel{5^x}\left(1+\left(\frac{3}{5}\right)^x\right)}{\cancel{5^x}\cdot x}$

$$\frac{1+\left(\frac{3}{5}\right)^{n\to\infty}}{n} > \frac{1+\left(\frac{3}{5}\right)^{n+1\to 0}}{n+1}$$

$|a_n| > |a_{n+1}|$

$|a_n|\downarrow \overset{\text{Л.К.}}{\Longrightarrow}$ УКВ

19. $\sum\limits_{n=1}^{\infty} \frac{2^n+n^3}{n^2}\cdot p^n$

$|a_n| = \left|\frac{2^n+n^3}{n^2}\cdot p^n\right| = \frac{2^n+n^3}{n^2}\cdot |p|^n$

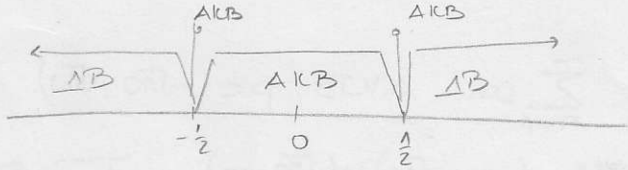
$\lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{2^n+n^3}{n^2}\cdot|p|^n} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{2^n}\cdot\sqrt[n]{1+\frac{n^3}{2^n}}\cdot\sqrt[n]{|p|^n}}{(\sqrt[n]{n})^2} = 2|p|$

$\xRightarrow{\text{К.К.}}$ $\sum\limits_{n=1}^{\infty}|a_n|$ КВ $2|p|<1$ $p\in(-\frac{1}{2},\frac{1}{2})$

ДВ $2|p|>1$ $p\in(-\infty,-\frac{1}{2})\cup(\frac{1}{2},\infty)$

? $2|p|=1$ $p=\pm\frac{1}{2}$

$\Rightarrow$ $\boxed{\sum\limits_{n=1}^{\infty} a_n \text{ АКВ } p\in(-\frac{1}{2},\frac{1}{2})}$



не КВА $p\in(-\infty,-\frac{1}{2})\cup(\frac{1}{2},\infty)$ $\boxed{\text{ДВ или УКВ}}$

$\underline{p\in(-\infty,-\frac{1}{2})\cup(\frac{1}{2},\infty)}$

$\sum\limits_{n=1}^{\infty}\frac{2^n+n^3}{n^2}\cdot p^n$

$\lim\limits_{n\to\infty}|a_n| = \lim\limits_{n\to\infty}\frac{2^n+n^3}{n^2}\cdot|p|^n$

$p>\frac{1}{2}$

$\lim\limits_{n\to\infty}\frac{2^n+n^3}{n^2}p^n = \lim\limits_{n\to\infty}\frac{(2p)^n\left(1+\frac{n^3}{2^n}\right)}{n^2} = \infty \neq 0$ $\Rightarrow a_n \underset{n\to\infty}{\not\to} 0$ ДВ

$p<-\frac{1}{2}$ $|a_n| \underset{n\to\infty}{\not\to} 0$ ДВ

$\boxed{p=\pm\frac{1}{2}}$

$\underline{p=\frac{1}{2}}$ $\sum\limits_{n=1}^{\infty}\frac{2^n+n^3}{2^n\cdot n^2}$

$a_n = \frac{2^n+n^3}{2^n\cdot n^2} = \frac{\cancel{2^n}(1+\frac{n^3}{2^n})}{\cancel{2^n}\cdot n^2} \sim \frac{1}{n^2} = b_n$

$\lim\limits_{n\to\infty}\frac{a_n}{b_n}=1$ и $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}$ $p>1$ КВ

$\xRightarrow{\text{п.к.}}$ $\sum\limits_{n=1}^{\infty} a_n$ АКВ

$\underline{p=-\frac{1}{2}}$ $\sum\limits_{n=1}^{\infty}\frac{2^n+n^3}{(-1)^n\cdot 2^n n^2}$

$|a_n| = \left|\frac{2^n+n^3}{(-1)^n\cdot 2^n n^2}\right| = \frac{2^n+n^3}{2^n\cdot n^2} = \frac{\cancel{2^n}(1+\frac{n^2}{2^n})}{\cancel{2^n}\cdot n^2} \sim \frac{1}{n^2}$ $p>1$

$\sum\limits_{n=1}^{\infty}|a_n|$ КВ $\Rightarrow \sum\limits_{n=1}^{\infty} a_n$ АКВ

(20) $\sum_{n=1}^{\infty} \frac{(p^2-9)^n}{2n+3}$

$$|a_n| = \frac{(|p^2-9|)^n}{2n+3}$$

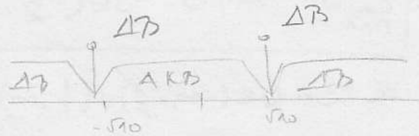
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{(|p^2-9|)^n}{2n+3}} = \lim_{n\to\infty} \frac{\sqrt[n]{(|p^2-9|)^n}}{\underbrace{\sqrt[n]{2n}}_{1}\,\underbrace{\sqrt[n]{1+\frac{3}{2n}}}_{\to 1}} = |p^2-9|$$

$\xRightarrow{\text{К.К}} \sum_{n=1}^{\infty} |a_n|$ КВ $|p^2-9|<1$ $p \in (-\sqrt{10}, \sqrt{10})$

ДВ $|p^2-9|>1$ $p \in (-\infty, -\sqrt{10}) \cup (\sqrt{10}, \infty)$

? $|p^2-9|=1$ $p = \pm\sqrt{10}$

$\Rightarrow \sum_{n=1}^{\infty} a_n$ АКВ $p \in (-\sqrt{10}, \sqrt{10})$



не КВА $(-\infty, -\sqrt{10}) \cup (\sqrt{10}, \infty)$

ДВ или УКВ

1. $\sum_{n=1}^{\infty} \frac{(p^2-9)^n}{2n+3}$

$p > \sqrt{10}$ $\lim_{n\to\infty} \frac{(p^2-9)^n}{2n+3} = \infty$ $a_n \underset{n\to\infty}{\not\to} 0$ ДВ

$p < -\sqrt{10}$ $a_n \underset{n\to\infty}{\not\to} 0$ ДВ

$p = \pm\sqrt{10}$

$\sum_{n=1}^{\infty} = \frac{1}{2n+3}$

$a_n = \frac{1}{2n+3} \sim \frac{1}{2n} = \frac{1}{2} \cdot \frac{1}{n}$ $\sum \frac{1}{n}$ ДВ

$\xRightarrow{\text{п.к}} \sum_{n=1}^{\infty} a_n$ ДВ

$p = -\sqrt{10}$

21) $\sum_{n=1}^{\infty} \frac{n}{(3n^2-2)^n(p^2-1)^n}$

$|a_n| = \frac{n}{(3n^2-2)^n(|p^2-1|)^n}$

$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{(3n^2-2)^n(|p^2-1|)^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}^{\to 1}}{\sqrt[n]{(3n^2-2)^n}\sqrt[n]{(|p^2-1|)^n}} = \lim_{n\to\infty} \frac{1}{3n^2\left(1-\frac{2}{3n^2}\right)|p^2-1|} = 0$

$\overset{КК}{\Longrightarrow} \sum_{n=1}^{\infty}|a_n|$ КВ $p\in\mathbb{R}$

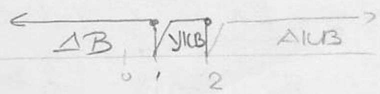
$\Longrightarrow \sum_{n=1}^{\infty} a_n$ АКВ $p\in\mathbb{R}$

22) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt[3]{n^3+1}}{n^p}$

$|a_n| = \left|(-1)^n \frac{\sqrt[3]{n^3+1}}{n^p}\right| = \frac{\sqrt[3]{n^3+1}}{n^p} = \frac{\sqrt[3]{n^3}\sqrt[3]{1+\frac{1}{n^3}}}{n^p} \sim \frac{1}{n^{p-1}}$ $\quad \sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ — КВ $p-1>1$ $p>2$; ДВ $p-1\le 1$ $p\le 2$

$\overset{П.К.}{\Longrightarrow} \sum_{n=1}^{\infty}$ — КВ $p>2$; ДВ $p\le 2$

$\Longrightarrow \sum_{n=1}^{\infty} a_n$ АКВ $p>2$



$p\le 2$

$|a_n| = \frac{\sqrt[3]{n^3+1}}{n^p} > 0$

не КВА $p\le 2$ (ДВ или УКВ)

1. $\lim_{n\to\infty} \frac{\sqrt[3]{n^3+1}}{n^p} = \lim_{n\to\infty} \frac{\cancel{n}\sqrt[3]{1+\frac{1}{n^3}}}{n^{p-1}} = \lim_{n\to\infty}\frac{1}{n^{p-1}}$ $\quad p-1>0$

$p-1\le 0$

$p\le 1$ ДВ

$p\in(1,2]$

2. $|a_n|\searrow$

$f(x) = \frac{\sqrt[3]{x^3+1}}{x^p}$

$f'(x) = \frac{\frac{1}{\cancel{3}}(x^3+1)^{-\frac{2}{3}}\cdot \cancel{3}x^2\cdot x^p - p x^{p-1}\sqrt[3]{x^3+1}}{x^{2p}} = \frac{\frac{1}{\sqrt[3]{(x^3+1)^2}}\cdot x^{p-1}\left(x^3 - p\cdot(x^3+1)\right)}{x^{2p}} =$

$= \frac{\underbrace{\frac{1}{\sqrt[3]{(x^3+1)^2}}\cdot x^{p-1}}_{>0}\underbrace{\left(x^3\underbrace{(1-p)}_{<0} - p\right)}_{\le 0}}{\underbrace{x^{2p}}_{>0}} < 0$

$\overset{Л.К}{\Longrightarrow} p\in(1,2]$ рез УКВ

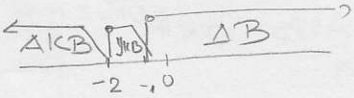
(23) $\sum_{n=1}^{\infty} (-1)^{n-1} n^p \sqrt{n^2+2n}$

$|a_n| = \left|(-1)^{n-1} \cdot n^p \sqrt{n^2+2n}\right| = n^p \cdot \sqrt{n^2+2n} = n^p \cdot n \sqrt{1+\frac{2}{n}} \sim n^{p+1} = \frac{1}{n^{-p-1}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{-p-1}}$ — КВ $\quad -p-1 > 1 \quad p < -2$
— ДВ $\quad -p-1 \le 1 \quad p \ge -2$

$\overset{\text{П.К}}{\Longrightarrow} \sum_{n=1}^{\infty} |a_n|$ — КВ $\quad p < -2$
— ДВ $\quad p \ge -2$

$\Longrightarrow \sum_{n=1}^{\infty} a_n$ — АКВ $\quad p < -2$
— не АКВ $\quad p \ge -2$



$p \ge -2$, $\quad a_n = n^p \sqrt{n^2+2n} > 0$

1. $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} n^p \cdot n \sqrt{1+\frac{2}{n}} = \lim_{n\to\infty} \frac{1}{n^{-p-1}}$

$-p-1 > 0$
$-p > 1$
$p < -1$

$\boxed{p \ge -1 \quad \text{ДВ}}$

$-2 \le p < -1$

2. $|a_n| \downarrow$

$f(x) = x^p \cdot \sqrt{x^2+2x}$

$f'(x) = p x^{p-1} \cdot \sqrt{x^2+2x} + x^p \cdot \frac{1}{\cancel{2}\sqrt{x^2+2x}} (\cancel{2}x + \cancel{2}) =$

$= \frac{x^{p-1}}{\sqrt{x^2+2x}} \left(p \cdot (x^2+2x) + x(x+1)\right) =$

$= \underbrace{\frac{x^{p-1}}{\sqrt{x^2+2x}}}_{>0} \underbrace{\left(p(x^2+2x) + (x^2+x)\right)}_{<0} < 0$

$x^2 + 2x > x^2 + x$

$|p|(x^2+2x) > x^2 + x$

$\overset{\text{П.К}}{\Longrightarrow} \sum_{n=1}^{\infty} a_n \quad -2 \le p < 1 \quad$ УКВ

24. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n^2+4n)^p}{\sqrt[3]{n+1}}$

$$|a_n| = \left| (-1)^{n-1} \frac{(n^2+4n)^p}{\sqrt[3]{n+1}} \right| = \frac{(n^2+4n)^p}{\sqrt[3]{n+1}} = \frac{n^{2p}\left(1+\frac{4}{n}\right)^p}{\sqrt[3]{n}\cdot\sqrt[3]{1+\frac{1}{n}}} \sim \frac{1}{n^{\frac{1}{3}-2p}}$$

$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}-2p}}$ — КВ $\frac{1}{3}-2p>1$ $\quad p<-\frac{1}{3}$

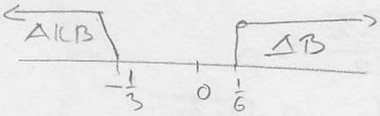
— ДВ $\frac{1}{3}-2p\leq 1$ $\quad p\geq -\frac{1}{3}$

$\overset{нк}{\Longrightarrow} \sum_{n=1}^{\infty} |a_n|$ — КВ $p<-\frac{1}{3}$

— ДВ $p\geq -\frac{1}{3}$

$\Longrightarrow \sum_{n=1}^{\infty}$ — АКВ $p<-\frac{1}{3}$

— не КВА $p\geq -\frac{1}{3}$



$p\geq -\frac{1}{3}$ $\quad a_n = \frac{(n^2+4n)^p}{\sqrt[3]{n+1}} > 0$

1. $\lim_{n\to\infty} \frac{(n^2+4n)^p}{\sqrt[3]{n+1}} = \lim_{n\to\infty} \frac{n^{2p}\left(1+\frac{4}{n}\right)^p}{\sqrt[3]{n}\ \sqrt[3]{1+\frac{1}{n}}} = \lim_{n\to\infty} \frac{1}{n^{-2p+\frac{1}{3}}}$

$-2p+\frac{1}{3}>0$

$2p<\frac{1}{3}$

$p<\frac{1}{6}$

$-\frac{1}{3}\leq p<\frac{1}{6}$ $\qquad p\geq\frac{1}{6}$ ДВ

2. $|a_n|\downarrow$ $\qquad (x+1)^{\frac{1}{3}}$

$f(x) = \frac{(x^2+4x)^p}{\sqrt[3]{x+1}}$

$$f'(x) = \frac{p(x^2+4x)^{p-1}(2x+4)\cdot\sqrt[3]{x+1} - (x^2+4x)^p\cdot\frac{1}{3}\frac{1}{\sqrt[3]{(x+1)^2}}}{\sqrt[3]{(x+1)^2}} =$$

$$= \frac{\frac{1}{3}(x^2+4x)^p\cdot\frac{1}{\sqrt[3]{(x+1)^2}}\left(3\cdot p\frac{(x+1)(2x+4)}{x^2+4x}-1\right)}{\sqrt[3]{(x+1)^2}} =$$

$(x+1)(x+2)$

$$= \frac{\frac{1}{3}(x^2+4x)^{p-1}\frac{1}{\sqrt[3]{(x+1)^2}}\left(6p\cdot\frac{x^2+3x+2}{x^2+4x}\right)}{\sqrt[3]{(x+1)^2}} =$$

25. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt[3]{n^3+2}}{n^{2p}}$

$$|a_n| = \left| (-1)^n \frac{\sqrt[3]{n^3+2}}{n^{2p}} \right| = \frac{\sqrt[3]{n^3+2}}{n^{2p}} = \frac{n\sqrt[3]{1+\frac{2}{n^3}}}{n^{2p}} \sim \frac{1}{n^{2p-1}} \quad u \sum_{n=1}^{\infty} \frac{1}{n^{2p-1}} \begin{cases} KB & 2p-1 > \\ \Delta B & 2p-1 \leq 1 \end{cases}$$

$$\xrightarrow{n.k.} \sum_{n=1}^{\infty} |a_n| \begin{cases} KB & p > 1 \\ \Delta B & p \leq 1 \end{cases}$$

$$\Longrightarrow \sum_{n=1}^{\infty} a_n \begin{cases} AKB & p > 1 \\ \cancel{K} \; KB\Delta & p \leq 1 \end{cases}$$

ΔB | YKB | AKB — 0, $\frac{1}{2}$, 1

$p \leq 1 \quad a_n = \frac{\sqrt[3]{n^3+2}}{n^{2p}} > 0$

1. $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n\sqrt{1+\frac{2}{n^3}}}{n^{2p}} = \lim_{n\to\infty} \frac{1}{n^{2p-1}}$

$2p-1 > 0$

$2p > 1$

$p > \frac{1}{2}$

$\Rightarrow \sum_{n=1}^{\infty} a_n \;\; \Delta B \;\; p \leq \frac{1}{2}$

$\boxed{\frac{1}{2} < p \leq 1}$

2. $|a_n| \searrow$

$$f(x) = \frac{\sqrt[3]{x^3+2}}{x^{2p}} = \frac{(x^3+2)^{\frac{1}{3}}}{x^{2p}}$$

$$f'(x) = \frac{\cancel{\frac{1}{3}}(x^3+2)^{-\frac{2}{3}} \cdot \cancel{3}x^2 \cdot x^{2p} - 2px^{2p-1} \cdot (x^3+2)^{\frac{1}{3}}}{x^{4p}} =$$

$$= \frac{x^{2p-1} \cdot (x^3+2)^{-\frac{2}{3}} \left(x^2 \cdot x - 2p(x^3+2) \right)}{x^{4p}} =$$

$$= \frac{x^{2p-1} \left(x^3(1-2p) - 4p \right)}{x^{4p} \sqrt[3]{(x^3+2)^2}} < 0$$

$1 < 2p \leq 2$

$(1-2p)^{<0}$

$\Downarrow$

$x^3(1-2p) < 0$

$x^3(1-2p) - 4 < 0$

$\xRightarrow{n.k.}$ pes YKB $p \in (\frac{1}{2}, 1]$

16. $\sum_{n=1}^{\infty} (-1)^{n-1} n^2 \sin \frac{\pi}{2^n}$

$a_n = \left| (-1)^{n-1} \cdot n^2 \sin \frac{\pi}{2^n} \right| = n^2 \sin \frac{\pi}{2^n} \sim n^2 \cdot \frac{\pi}{2^n}$